11 Important Ideas from Probability Theory

1. **frequentist** understanding of probability/chance: long run fraction/ percentage of times an event happens when a process is repeated again and again and … Examples: fair coin, biased coin, fair die, biased die, 3 card deck, full deck, bingo wheel, compound events (even number for die, both black and white die show 1)

2. We can write chances as fractions (e.g., 1/5) or as percents (e.g., 20%). If you need to multiply two percentages (e.g., the multiplication rule below), then use fractions. To see why remember that 20% mathematically means 50 per 100 or 50/100. If we multiply 50/100 x 50/100, we get 2,500/10,000 = 25%. We **do not** get 50% x 50% = 250%. It turns out that we can add percents, because 50/100 + 50/100 = 100/100.

3. chance of the opposite of an event is just 100% - ch. of event (as a percent) (or 1 - chance of an event as a fraction). Examples: fair die, Bingo wheel, 52-card deck.

4. **conditional probabilities**: realize that something might have changed from the original setup (a card has been drawn, etc.). Key word is "given." Examples: 3 card deck, black and white dice, 52 card deck, bingo wheel.

5. **multiplication rule**: chance that two things both happen ("AND!") is the chance (as a fraction) of the first times the conditional chance (as a fraction) of the second, given that the first already happened. Examples: 3 card deck, black and white dice, 52 card deck, bingo wheel.

6. **independence**: two things are independent if the chances of the second event are the same, no matter how the first turns out. Examples: 1st and 2nd draw from 3 card deck, black and white dice, 1st and 2nd balls from the Bingo wheel.

7. When drawing with replacement, the events are independent. When drawing without replacement, the events are dependent.

8. To calculate the chance of a complicated process, try to write down the entire list of equally likely outcomes. Examples: What's the chance of getting a red card from a 52-card deck? What's the chance of the black and white die summing to 7?

9. **mutually exclusive**: two events are mutually exclusive if the cannot both happen. One prevents that occurrence of the other. One excludes the other. If one happens, the other cannot. Examples: six with black, six with white; six with white, five with white; red with first draw, red with second draw; 1st and 2nd draw from 3-card deck.

10. **addition rule**: The chance that at least one of two things happens is the sum of the two chances (as a fraction or percent), provided the two events are mutually exclusive.

    1. Example: What's the chance of rolling an even number with a die?

    2. Example: What's the chance of tossing a head OR a tail?

    3. Example: What's the chance of drawing a Queen from a 52-card deck?

    4. What the chance of rolling at least one six with the black and white dice?

11. It's sometimes easier (especially if you need the addition rule, but events aren't mutually exclusive) to compute the chance of the opposite (and then subtract hat percent from 100%). Example: What the chance of rolling at least one six with the black and white dice? This is the opposite of rolling a not-six with both dice.