Expected Value & Standard Error

What does the average from the NTM look like?

Numbered-Ticket Model

… *is like* drawing ____ times from the box ___________ with replacement and averaging the draws.

Example #1

Rolling a die 40 times and averaging the numbers shown

…is like…

drawing ____ times from the box ______________ with replacement and averaging the draws.

Numbered-Ticket Model

… *is like* drawing ____ times from the box ____________ with replacement and averaging the draws.

Example #2

Rolling a die 40 times and computing the proportion of aces

…is like…

drawing ____ times from the box ______________ with replacement and averaging the draws.

Fact

If we execute the box model, the result is an <u>average</u>.

Numbered-Ticket Model

… *is like* drawing ____ times from the box __________ with replacement and averaging the draws.

Question

What can we say about this (yet to be produced) average?

Question

What can we say about this (yet to be produced) average?

the expected value

the standard error

The average will be about ____ give or take ____ or so.

Suppose an <u>actual list</u> of numbers.

The entries in that list are about [the average] give or take [the SD] or so.

Suppose a <u>hypothetical list</u> of numbers that we generate by executing the NTM an infinite number of times.

The (hypothetical, long-run) average is the expected value.

The (hypothetical, long-run) SD is the standard error.

We can think of the expected value and standard error as a "long-run" average and SD of a chance process.

	actual list of numbers	**chance process**
typical value	average	expected value
give or take	SD	standard error

Equations

$$\textbf{expected value for average} = \textbf{average of tickets in the box}$$

$$\textbf{SE for average} = \frac{\textbf{SD of tickets in the box}}{\sqrt{\textbf{number of draws}}}$$

Helpful Hints

Suppose the tickets are "big-small" so that each ticket is either big B or small S (e.g., the tickets 2, 2, 2, 2, 14, 14), then

$$\textbf{SD of big-small tickets} = (\mathbf{B} - \mathbf{S}) \times \sqrt{(\textbf{fraction that are B}) \times (\textbf{fraction that are S})}$$

Suppose the are "0-1" so that each ticket is either 0 or 1 (e.g., the box 0, 0, 0, 1), then

$$\textbf{SD of 0-1 tickets} = \sqrt{(\textbf{fraction that are 0}) \times (\textbf{fraction that are 1})}$$

Example #1

Rolling a die 40 times and averaging the numbers shown

...is like...

drawing __40__ times from the box __1, 2, 3, 4, 5, 6__ with replacement and averaging the draws.

the expected value the standard error

The avg will be about __3.5__ give or take _____ or so.

expected value for average = average of tickets in the box

= 3.5

Example #1

Rolling a die 40 times and averaging the numbers shown

...is like...

drawing 40 times from the box 1, 2, 3, 4, 5, 6 with replacement and averaging the draws.

the expected value

the standard error

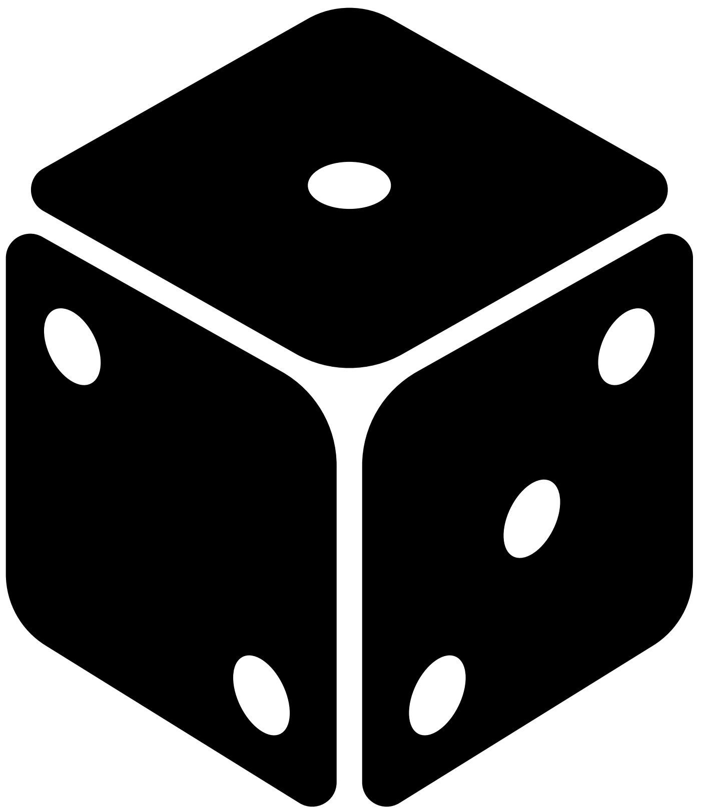
The sum will be about 3.5 give or take 0.27 or so.

$$\text{SE for average} = \frac{\text{SD of tickets in the box}}{\sqrt{\text{number of draws}}} = \frac{???}{\sqrt{40}} = \frac{1.71}{6.32} = 0.27$$

```
> x <- c(1, 2, 3, 4, 5, 6)
> sqrt(mean((x - mean(x))^2))
[1] 1.707825
```

I rolled a die 40 times and got an average of 3.53. That's right in line with our claim that the average will be about 3.5 give or take 0.27 or so.

I did it nine more times and got 2.90, 3.43, 3.58, 146, 3.65, 3.15, 3.40, 3.20, and 3.45. Again, that's right in line with our claim that the average will be about 3.5 give or take 0.27 or so.



Example #3

Tossing a coin 100 times and computing the proportion of heads

...is like...

drawing 100 times from the box 0, 1 with replacement and averaging the draws.

the expected value

the standard error

The average will be about 0.5 give or take 0.05 or so.

$$\text{SE for average} = \frac{\text{SD of tickets in the box}}{\sqrt{\text{number of draws}}} = \frac{\sqrt{0.5 \times 0.5}}{\sqrt{100}} = \frac{0.5}{10} = 0.05$$

trick: SD of a 0-1 box = sqrt(frac 1s × frac 0s)

Two things to notice:

1. The SD of a 0-1 box is EASY!
2. The SD of the box {0, 1} (i.e., a single 0 and a single 1) equals 0.5.

the expected value

the standard error

The average will be about 0.5 give or take 0.05 or so.

0.44 0.46

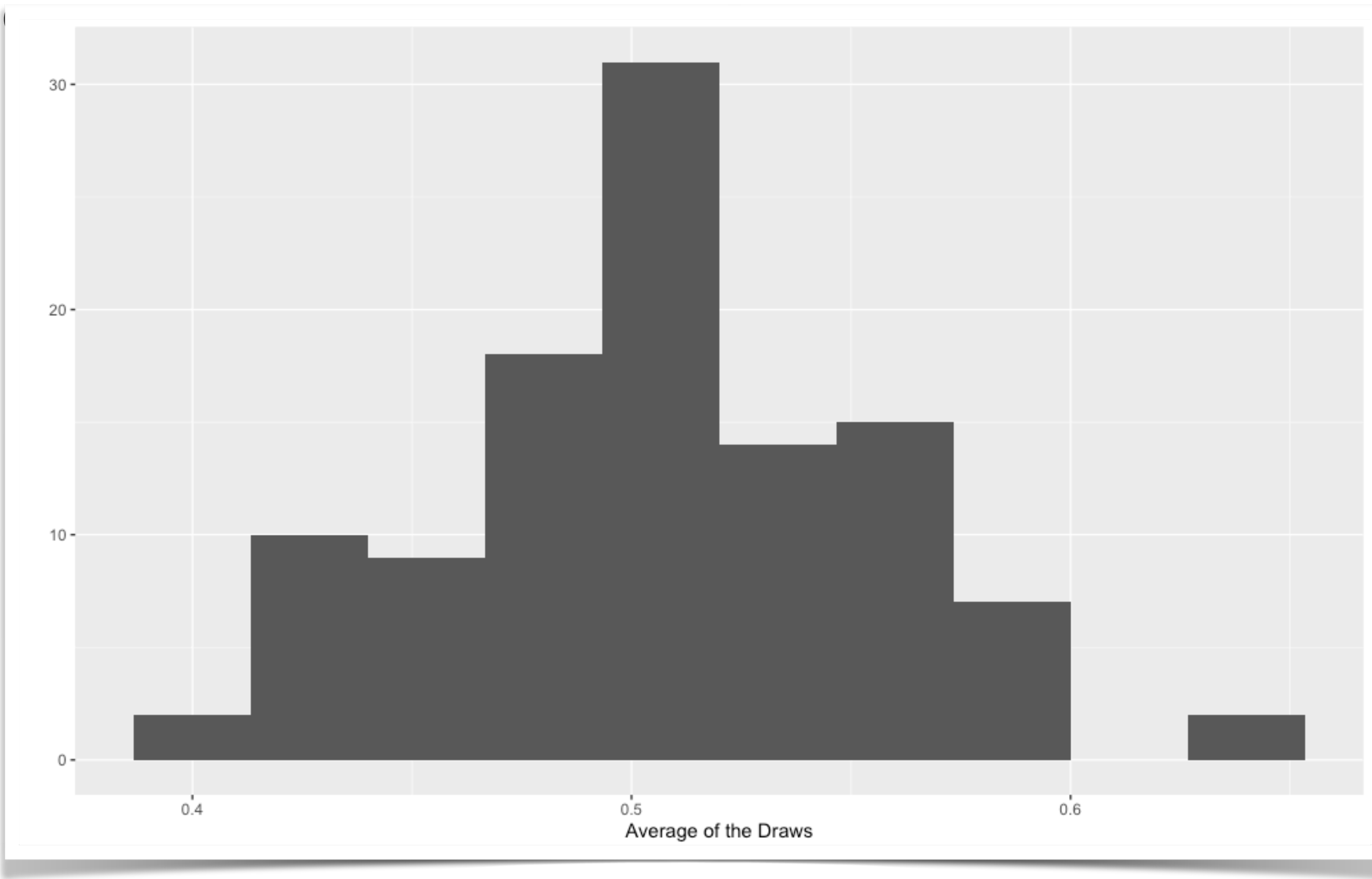
0.53 0.55

0.51 0.49

0.55 0.52

0.50 0.49

0.44 0.48



0.56 0.53

0.48 0.48

0.44 0.45

0.37 0.48

0.42 0.59

0.55 0.48

Highlights, Again

- If we have a chance process, we can sometimes describe it with a **numbered-ticket model**.
- If we have a box model, then we compute the **expected value** and the **standard error** for the sum:

$$\textbf{expected value for average} = \textbf{average of box}$$

and

$$\textbf{SE for average} = \frac{\textbf{SD of box}}{\sqrt{\textbf{number of draws}}}.$$

- With the expected value and standard error, we can fill in the following: **The average will be about ___ give or take ___ or so.**
- The SD of 0-1 box (i.e., a box with only 0s and 1s) has an EASY formula:

$$\textbf{SD of 0-1 box} = \sqrt{(\textbf{fraction that are 0}) \times (\textbf{fraction that are 1})}.$$

- The SD of the box {0, 1} (i.e., a single 0 and a single 1) is $\sqrt{(0.5) \times (0.5)} = 0.5$